JAP-003-001105 Seat No. ________

B. Sc. (Sem. I) (CBCS) Examination

November - 2019

M - 101 : Geometry & Calculus

(Old Course)

Faculty Code : 003

Subject Code : 001105

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Right hand side digit indicate the marks.

1 Answer the following questions in short : **20**

(1) Find the polar form of the equation $x^2 + y^2 = 9$.

(2) Write the equation of a sphere having center (a, b, c) and radius r.

(3) The Cartesian coordinate of polar point $(2, \pi)$ is ________

(4) Find the center and radius of the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 1 = 0$.

(5) If $y = \sin(x)$, then find $\frac{d^n y}{d\,x^n}$.

(6) $\frac{d^5 x^5}{d\,x^5} =$ ________

(7) The set $\mathbb{N}$ is lower bounded. (True/False)

(8) The function $f(x) = 6(x-2)^2, x < 2$ is ________ (Increasing / Decreasing)

(9) Evaluate $\lim\limits_{x \to 0} \frac{\sin x}{x \cos x}$.

(10) Write the indeterminate form of $\lim\limits_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.

(11) Find the integrating factor of the differential equation $\frac{dy}{dx} = x + y$.

(12) Write Clairaut's differential equation.

(13) If m_1, m_2 are distinct real roots and m_3, m_4 are equal real roots of auxiliary equation of homogeneous differential equation, then write the solution.

(14) $\frac{1}{D}x^3 =$ __________

(15) $(1+D)^{-1} =$ __________

(16) Find $\int_0^{\frac{\pi}{2}} \cos^{10}(x)\,dx.$

(17) Evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^4 x\,dx.$

(18) $\frac{1}{f(D^2)}\cos ax =$ __________, provided $f(-a^2) \neq 0.$

(19) Define first order first degree homogeneous differential equation.

(20) State the alternative form of Lagrange's mean value theorem.

2 (A) Attempt any **three** out of six : **6**

(1) Find the path of the point which is at the distance 3 from the polar point $\left(5, \frac{\pi}{2}\right)$.

(2) Find the equation of a sphere having $A(2,-3,4)$ and $B(-2,3,-4)$ as extemities of diameter.

(3) Derive the nth derivative of $\cos(ax+b)$.

(4) State Cauchy's mean value theorem and derive Lagrange's mean value theorem from it.

(5) Show that Maclaurin's series expansion of e^x is $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots,$ for all $x \in \mathbb{R}$.

(6) If $f(x) = \frac{x\cos x - \log(x+1)}{x^2}$, then show that $\lim_{x\to 0} f(x) = \frac{1}{2}.$

(B) Attempt any **three** out of six : **9**

(1) Find the equation of the circle in polar coordinate system whose tangent is the initial line.

(2) Find approximate value of $\log_{10} 73.55$, correct up to 6 decimal places, where $\log_{10} 73 = 1.863323$, $\log_{10} e = 0.43429$.

(3) Derive the condition for the equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ to be a sphere.

(4) If $y = e^{ax}\sin(bx+c)$, then find $\dfrac{d^n y}{d\,x^n}$.

(5) Show that $\lim\limits_{x\to a} \dfrac{\log(x-a)}{\log\left(e^x - e^a\right)} = 1$.

(6) Verify Lagrange's mean value theorem for the function $f(x) = x^2 + x$, for $x \in [0, 1]$.

(C) Attempt any **two** out of five : **10**

(1) Prove that the plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius unity and find the equation sphere which has this circle for one of the great circles.

(2) State and prove Libnitz's theorem.

(3) State and prove Roll's mean value theorem.

(4) Show the $e^{\sin^{-1}x} = 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \ldots.$

(5) Show that $\dfrac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \dfrac{v-u}{1+u^2}, 0 < u < v$ and deduct $\dfrac{\pi}{4} + \dfrac{3}{25} < \tan^{-1} < \dfrac{4}{3}\dfrac{\pi}{4} + \dfrac{1}{6}$.

3 (A) Attempt any **three** out of six : **6**

(1) Solve $\left(D^4 + 2D^2 + 1\right)y = 0$.

(2) Solve $\dfrac{dy}{dx} + x\sin(2y) = x^3\cos^2 y$.

(3) Solve $\left(x^2 - ay\right)dx + \left(y^2 - ax\right)dy = 0.$

(4) Solve $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = e^{2x}.$

(5) Find particular integral of $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log(x)}{x^2}.$

(6) Find the general solution of $(y - px)(p - 1) = 0.$

(B) Attempt any **three** out of six : **9**

(1) Solve $\frac{d^2y}{dx^2} - 11\frac{dy}{dx} + 18y = e^{7x}.$

(2) Solve $x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5.$

(3) Solve $x^2(y - px) = yp^2.$

(4) Evalaute $\int_0^a \frac{x^7}{\sqrt{\left(a^2 - x^2\right)}}dx.$

(5) Solve $y - 2px = \tan^{-1}\left(xp^2\right).$

(6) Find particular integral of $\left(D^2 + 9\right)y = e^{2x} + 2x.$

(C) Attempt any **two** out of five : **10**

(1) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2\log(x).$

(2) Solve $x^2\frac{d^2y}{dx^2} + y = 3x^2.$

(3) Find the solution first order linear differential equation.

(4) Find the reduction formula for $\int \sin^m x \cdot con^n x dx,$ where $m, n \in \mathbb{N}$

(5) State and prove the necessary and sufficient condition for the differential equation $M(x, y)dx + N(x, y)dy = 0$ to be exact.
